



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

157. Proposed by A. H. HOLMES, Brunswick, Maine.

Find integral values for m and n in $64m^2n^2(m^2-n^2)^2 + (m^2+n^2)^4 = \square$.

No solution of this problem has been received.

158. Proposed by J. EDWARD SANDERS, Weather Bureau, Chicago, Ill.

Find positive rational values of a and b in the equation $x^4 - 2ax^2 + x + a^2 - b = 0$, that will make each of the roots (all different) rational numbers.

Solution by E. B. ESCOTT, Ann Arbor, Mich.

Let two roots be $x=a$ and $x=\beta$. Substituting, we have

$$b = a^4 - 2a^2 + a + a^2 = \beta^4 - 2a\beta^2 + \beta + a^2.$$

Transposing, and removing factor $a-\beta$ (since $a \neq \beta$), we have $a^3 + a^2\beta + a\beta^2 + \beta^3 - 2a(a+\beta) + 1 = 0$, i. e.,

$$a = \frac{1}{2} \left(a^2 + \beta^2 + \frac{1}{a+\beta} \right).$$

Substituting, we get $b = \frac{1}{4} \left(a^2 - \beta^2 - \frac{1}{a+\beta} \right)^2 + a$.

Substituting in the original equation, and removing the factors $x-a$ and $x-\beta$, we have

$$x^2 + (a+\beta)x + \left(a\beta - \frac{1}{a+\beta} \right) = 0.$$

This will have commensurable roots if

$$(a+\beta)^2 - 4 \left(a\beta - \frac{1}{a+\beta} \right) = (a-\beta)^2 + \frac{4}{a+\beta} = r^2.$$

It is easily seen that a and β cannot be integral.

We can get as many rational values as we please by assuming any value for $a+\beta$.

Example. Let $a+\beta = \frac{1}{2}$. Then $\beta = \frac{1}{2} - a$. Substituting in the last equation, we get $(2a - \frac{1}{2})^2 + 8 = r^2$. This can be satisfied in an infinite number of ways, e. g.,

$$2a - \frac{1}{2} + r = 4, \quad 2a - \frac{1}{2} - r = -2,$$